

AD-A102 160

CALIFORNIA UNIV BERKELEY OPERATIONS RESEARCH CENTER

F/6 12/2

MULTIPLE RESOURCE SMOOTHING IN SHIPYARD PLANNING.(U)

JUN 81 R C LEACHMAN, J BOYSEN

N00014-76-C-0134

UNCLASSIFIED

ORC-81-15

NL

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867
868
869
870
871
872
873
874
875
876
877
878
879
880
881
882
883
884
885
886
887
888
889
890
891
892
893
894
895
896
897
898
899
900
901
902
903
904
905
906
907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010
1011
1012
1013
1014
1015
1016
1017
1018
1019
1020
1021
1022
1023
1024
1025
1026
1027
1028
1029
1030
1031
1032
1033
1034
1035
1036
1037
1038
1039
1040
1041
1042
1043
1044
1045
1046
1047
1048
1049
1050
1051
1052
1053
1054
1055
1056
1057
1058
1059
1060
1061
1062
1063
1064
1065
1066
1067
1068
1069
1070
1071
1072
1073
1074
1075
1076
1077
1078
1079
1080
1081
1082
1083
1084
1085
1086
1087
1088
1089
1090
1091
1092
1093
1094
1095
1096
1097
1098
1099
1100
1101
1102
1103
1104
1105
1106
1107
1108
1109
1110
1111
1112
1113
1114
1115
1116
1117
1118
1119
1120
1121
1122
1123
1124
1125
1126
1127
1128
1129
1130
1131
1132
1133
1134
1135
1136
1137
1138
1139
1140
1141
1142
1143
1144
1145
1146
1147
1148
1149
1150
1151
1152
1153
1154
1155
1156
1157
1158
1159
1160
1161
1162
1163
1164
1165
1166
1167
1168
1169
1170
1171
1172
1173
1174
1175
1176
1177
1178
1179
1180
1181
1182
1183
1184
1185
1186
1187
1188
1189
1190
1191
1192
1193
1194
1195
1196
1197
1198
1199
1200
1201
1202
1203
1204
1205
1206
1207
1208
1209
1210
1211
1212
1213
1214
1215
1216
1217
1218
1219
1220
1221
1222
1223
1224
1225
1226
1227
1228
1229
1230
1231
1232
1233
1234
1235
1236
1237
1238
1239
1240
1241
1242
1243
1244
1245
1246
1247
1248
1249
1250
1251
1252
1253
1254
1255
1256
1257
1258
1259
1260
1261
1262
1263
1264
1265
1266
1267
1268
1269
1270
1271
1272
1273
1274
1275
1276
1277
1278
1279
1280
1281
1282
1283
1284
1285
1286
1287
1288
1289
1290
1291
1292
1293
1294
1295
1296
1297
1298
1299
1300
1301
1302
1303
1304
1305
1306
1307
1308
1309
1310
1311
1312
1313
1314
1315
1316
1317
1318
1319
1320
1321
1322
1323
1324
1325
1326
1327
1328
1329
1330
1331
1332
1333
1334
1335
1336
1337
1338
1339
1340
1341
1342
1343
1344
1345
1346
1347
1348
1349
1350
1351
1352
1353
1354
1355
1356
1357
1358
1359
1360
1361
1362
1363
1364
1365
1366
1367
1368
1369
1370
1371
1372
1373
1374
1375
1376
1377
1378
1379
1380
1381
1382
1383
1384
1385
1386
1387
1388
1389
1390
1391
1392
1393
1394
1395
1396
1397
1398
1399
1400
1401
1402
1403
1404
1405
1406
1407
1408
1409
1410
1411
1412
1413
1414
1415
1416
1417
1418
1419
1420
1421
1422
1423
1424
1425
1426
1427
1428
1429
1430
1431
1432
1433
1434
1435
1436
1437
1438
1439
1440
1441
1442
1443
1444
1445
1446
1447
1448
1449
1450
1451
1452
1453
1454
1455
1456
1457
1458
1459
1460
1461
1462
1463
1464
1465
1466
1467
1468
1469
1470
1471
1472
1473
1474
1475
1476
1477
1478
1479
1480
1481
1482
1483
1484
1485
1486
1487
1488
1489
1490
1491
1492
1493
1494
1495
1496
1497
1498
1499
1500
1501
1502
1503
1504
1505
1506
1507
1508
1509
1510
1511
1512
1513
1514
1515
1516
1517
1518
1519
1520
1521
1522
1523
1524
1525
1526
1527
1528
1529
1530
1531
1532
1533
1534
1535
1536
1537
1538
1539
1540
1541
1542
1543
1544
1545
1546
1547
1548
1549
1550
1551
1552
1553
1554
1555
1556
1557
1558
1559
1560
1561
1562
1563
1564
1565
1566
1567
1568
1569
1570
1571
1572
1573
1574
1575
1576
1577
1578
1579
1580
1581
1582
1583
1584
1585
1586
1587
1588
1589
1590
1591
1592
1593
1594
1595
1596
1597
1598
1599
1600
1601
1602
1603
1604
1605
1606
1607
1608
1609
1610
1611
1612
1613
1614
1615
1616
1617
1618
1619
1620
1621
1622
1623
1624
1625
1626
1627
1628
1629
1630
1631
1632
1633
1634
1635
1636
1637
1638
1639
1640
1641
1642
1643
1644
1645
1646
1647
1648
1649
1650
1651
1652
1653
1654
1655
1656
1657
1658
1659
1660
1661
1662
1663
1664
1665
1666
1667
1668
1669
1670
1671
1672
1673
1674
1675
1676
1677
1678
1679
1680
1681
1682
1683
1684
1685
1686
1687
1688
1689
1690
1691
1692
1693
1694
1695
1696
1697
1698
1699
1700
1701
1702
1703
1704
1705
1706
1707
1708
1709
1710
1711
1712
1713
1714
1715
1716
1717
1718
1719
1720
1721
1722
1723
1724
1725
1726
1727
1728
1729
1730
1731
1732
1733
1734
1735
1736
1737
1738
1739
1740
1741
1742
1743
1744
1745
1746
1747
1748
1749
1750
1751
1752
1753
1754
1755
1756
1757
1758
1759
1760
1761
1762
1763
1764
1765
1766
1767
1768
1769
1770
1771
1772
1773
1774
1775
1776
1777
1778
1779
1780
1781
1782
1783
1784
1785
1786
1787
1788
1789
1790
1791
1792
1793
1794
1795
1796
1797
1798
1799
1800
1801
1802
1803
1804
1805
1806
1807
1808
1809
1810
1811
1812
1813
1814
1815
1816
1817
1818
1819
1820
1821
1822
1823
1824
1825
1826
1827
1828
1829
1830
1831
1832
1833
1834
1835
1836
1837
1838
1839
1840
1841
1842
1843
1844
1845
1846
1847
1848
1849
1850
1851
1852
1853
1854
1855
1856
1857
1858
1859
1860
1861
1862
1863
1864
1865
1866
1867
1868
1869
1870
1871
1872
1873
1874
1875
1876
1877
1878
1879
1880
1881
1882
1883
1884
1885
1886
1887
1888
1889
1890
1891
1892
1893
1894
1895
1896
1897
1898
1899
1900
1901
1902
1903
1904
1905
1906
1907
1908
1909
1910
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925
1926
1927
1928
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
1941
1942
1943
1944
1945
1946
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032
2033
2034
2035
2036
2037
2038
2039
2040
2041
2042
2043
2044
2045
2046
2047
2048
2049
2050
2051
2052
2053
2054
2055
2056
2057
2058
2059
2060
2061
2062
2063
2064
2065
2066
2067
2068
2069
2070
2071
2072
2073
2074
2075
2076
2077
2078
2079
2080
2081
2082
2083
2084
2085
2086
2087
2088
2089
2090
2091
2092
2093
2094
2095
2096
2097
2098
2099
2100
2101
2102
2103
2104
2105
2106
2107
2108
2109
2110
2111
2112
2113
2114
2115
2116
2117
2118
2119
2120
2121
2122
2123
2124
2125
2126
2127
2128
2129
2130
2131
2132
2133
2134
2135
2136
2137
2138
2139
2140
2141
2142
2143
2144
2145
2146
2147
2148
2149
2150
2151
2152
2153
2154
2155
2156
2157
2158
2159
2160
2161
2162
2163
2164
2165
2166
2167
2168
2169
2170
2171
2172
2173
2174
2175
2176
2177
2178
2179
2180
21

LEVEL II

ORC 81-15
JUNE 1981

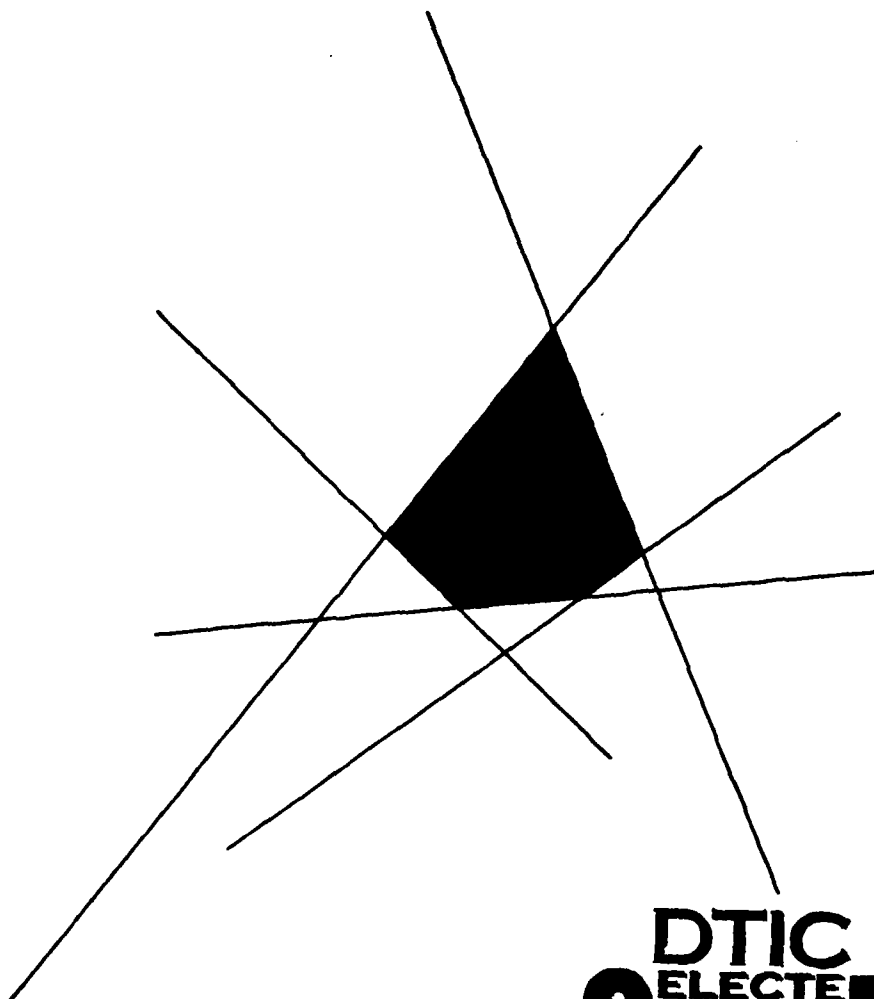
(12)

45

AD A102160

MULTIPLE RESOURCE SMOOTHING IN SHIPYARD PLANNING

by
ROBERT C. LEACHMAN
and
JOERG BOYSEN



OPERATIONS
RESEARCH
CENTER

DTIC
ELECTE
JUL 29 1981
S F D

DTIC FILE COPY

UNIVERSITY OF CALIFORNIA • BERKELEY

81 7 29 038


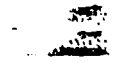
MULTIPLE RESOURCE SMOOTHING IN SHIPYARD PLANNING

by

Robert C. Leachman
Department of Industrial Engineering
and Operations Research
University of California, Berkeley

and

Joerg Boysen
Operations Research Center
University of California, Berkeley

Accession For	
NHLI GRA&I	
NHLI TAR	
Classified	
Classification	
Priority	
Priority Codes	
Serial and/or	
Special	
A	

JUNE 1981

ORC 81-15

This research was supported by the Office of Naval Research under Contract N00014-76-C-0134 with the University of California. Reproduction in whole or in part is permitted for any purpose of the United States Government.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ORC-81-15	2. GOVT ACCESSION NO. AD-A102 160	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MULTIPLE RESOURCE SMOOTHING IN SHIPYARD PLANNING		5. TYPE OF REPORT & PERIOD COVERED Research Report
7. AUTHOR(s) Robert C. Leachman and Joerg Boysen		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Operations Research Center University of California Berkeley, California 94720		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0134
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Department of the Navy Arlington, Virginia 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 047 033
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 71		12. REPORT DATE June 1981
		13. NUMBER OF PAGES 29
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Resource Leveling Critical Path Analysis Capacity Planning Nonlinear Programming Construction Systems Manpower Allocation		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (SEE ABSTRACT)		

DD FORM 1473
1 JAN 73

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ABSTRACT

✓ A heuristic approach for planning multiple resource workloads in project networks is discussed, with particular attention to ship overhaul planning. Projects are modeled using a critical-path-analysis activity network. Activity resource requirements are taken as given and activity loading is assumed to be constant. Activity duration variables are defined which then determine the demand rates per resource per activity.

67 An iterative nonlinear programming procedure assigns activity durations (and consequently, activity resource demand rates) to minimize resource peaking subject to meeting project due dates. Computational experience is described for application to a major ship overhaul.

iii

Table of Contents

	Page
I. Introduction and Scope of Problem.....	1
II. Construction Project Model	5
III. Resource Leveling.....	6
A. Peak Pricing Procedure	6
B. Optimization Problem Formulations.....	8
C. Procedure Termination and Cycling.....	11
D. Capacity Requirements vs. Project Delay	13
IV. Applications and Computational Experience.....	14
A. Network and Data.....	14
B. Experiments.....	14
V. Summary and Analysis of Results.....	22
Glossary of Notation	23
References.....	24

List of Exhibits

	Page
1. Multi-Overhaul Scheduling Chart.....	2
2. Example of Key-op Activity	3
3. Resource Peak Intervals	7
4. (a) Single-Element Slack Paths	10
(b) Redundant Slack Paths.....	10
5. Optimization Problem Constraint Sets	12
6. List of Labor Shops	15
7. Example of Aggregate Activity.....	16
8. Example of Shop Interaction	17
9. Experimental Results: Shop Capacity Requirements.....	19
10. Experiment (b): Sequence of Load Histories, Shop 5.....	20
11. Comparison of Shop 8 Load Histories, Experiments (b), (c), (d).....	21

I. Introduction and Scope of the Problem

Production activity in a naval shipyard consists of a variety of large ship overhaul projects. Each of these projects requires labor input in the tens or even hundreds of thousands of man-days from among 20 different trade shops. Project durations range from 3 months to 2½ years. Manpower loading on a ship overhaul is typically very peaked, and there is competition among various projects for manpower, dry docks, and other key resources. The planning and scheduling of projects so as to meet target completion dates while maintaining high levels of yard productivity is a challenging management problem.

In current practice, the problem is decomposed into two stages. First, an aggregate scheduling effort is undertaken, in which the "rough" scheduling of projects is made. In this effort, starting, ending, and a few intermediate *milestone* dates are established for each project. The goal of aggregate scheduling is to determine milestone dates which are timely but feasible, considering projections of available production manpower and dry docks.

Detailed information regarding resource and duration requirements for each project would be desirable to accomplish aggregate scheduling. But only very aggregate, preliminary, information is currently available to planners. A time history of the total man-days per working day is estimated from past experience for each project; by setting start dates for each project, these histories may be summed to estimate the total man-day history on the yard. By trial and error, milestone dates are developed for which the total load history is comparable to the projected available work force, and dry dock conflicts are avoided. (See Exhibit 1.) This load information is not broken down by labor shop, so that shop load infeasibilities may still arise.

In the second stage, detailed project scheduling of small component production activities known as *key-ops* is undertaken. The specification of these activities, numbering in the thousands for each overhaul, may require a year or more. A critical path network is then developed consisting of the key-op activities, which are then scheduled using critical path methods (CPM). In the scheduling effort network slack is uniformly allocated among activities, which tends to reduce the project's peak resource loads. See [3] for a discussion of CPM and resource load leveling methods. See Exhibit 2 for an example of a key-op description.

Milestone dates and key-op schedules are distributed to line (shop) production management, which is responsible for the day-to-day allocation of working crews and job supervision. In practice, adherence to key-op schedules is rare, but observance of milestone dates is accomplished where feasible. In a sense, line management accomplishes its own version of resource leveling by varying crew sizes and job assignments.

It is our opinion that aggregate scheduling techniques utilizing *more* detail than now used, coupled with scheduling techniques for individual projects based on *less* detailed activity

EXHIBIT 1
MULTI-OVERHAUL SCHEDULING CHART

PROJECTS	MONTHLY PROJECT LOADS				
	FEB	MAR	APR	MAY	JUN
PROJECT A	2031	1648	1289	1048	854
PROJECT B	2059	1896	1182	856	539
PROJECT C	1488	1759	2126	1652	1467
PROJECT D			234	1175	2077
OTHER	410	390	380	370	370
TOTAL LOAD	5988	5693	5211	5101	5307
WORKFORCE	4975	4906	4698	4743	4486
OVERLOAD/ (UNDERLOAD)	1013	787	513	358	821

ALL FIGURES ARE MAN-DAY TOTALS,
IRRESPECTIVE OF LABOR TYPE.

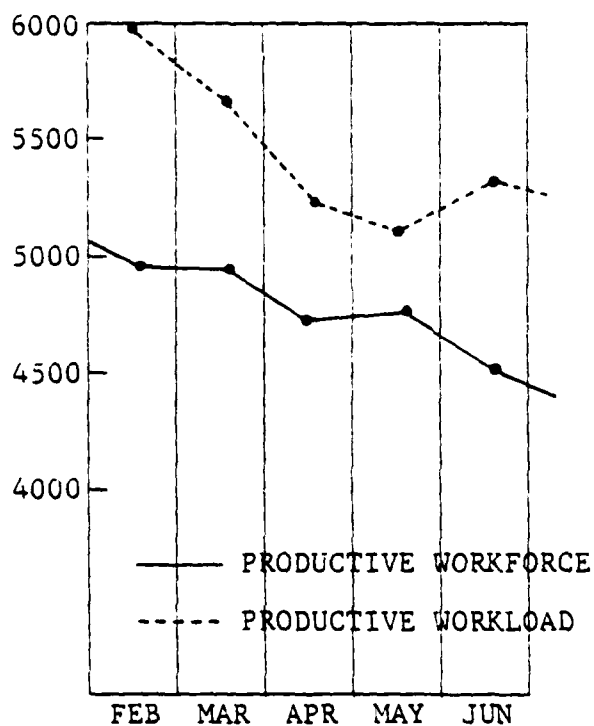


EXHIBIT 2

EXAMPLE OF KEY-OP ACTIVITY

ACTIVITY 16662-26301-603 INSTALL SHAFT LUBE PUMPS

DURATION: 15 DAYS

SHOP HOURS REQUIRED

SHOP 5 (SHIPBOARD MECHANICAL)	64 MAN HOURS
SHOP 7 (ELECTRICAL CABLES)	8 MAN HOURS
SHOP 8 (PIPEFITTERS)	48 MAN HOURS

4

definition than now pursued, would be valuable. Shop capacities should be explicitly treated in aggregate scheduling. Work scheduling should not be done at the *key-op level of detail*, since such fine scheduling is largely ignored in actual operations. An Operations Research model used in shipyard project scheduling should assign work in components more consistent with the schedule constraints perceived by line management. It should also reflect management's flexibility to vary crew sizes assigned to activities, as opposed to the mere scheduling of component activities, in efforts to level resource loads.

The development of improved aggregate scheduling techniques is deferred to a subsequent paper. In this paper, new techniques for individual project scheduling in a shipyard, motivated by the above discussion, are investigated.

II. Construction Project Model

The model presented here was originally formulated in [2] and [4]. We present a simplified version of it. The construction project is modeled as a collection of component activities A utilizing a set K of exogenous resources. The dependencies among activities are represented by an activity-on-arc CPM network. Given are a_k , the required amount of resource $k \in K$ to complete activity $i \in A$, and $d_i \geq 0$, the minimum duration of activity $i \in A$. Let N , a subset of A , be the set of activities requiring non-zero resources, i.e., $a_k > 0$ for some $k \in K$. Instantaneous application of resources is not possible, hence $d_i > 0$ for $i \in N$. In the following sections, activities are assumed to belong to A , and resources to K , unless otherwise indicated.

Let $d \equiv (d_1, \dots, d_q)$ and $t \equiv (t_1, \dots, t_q)$, where t is a vector of time assignment variables such that $t \geq d$. The rate of application of resource k to activity $i \in N$, a_k/t_i , is modeled as constant between the start and finish times of activity i . This represents a constant crew assignment to each activity $i \in N$. The crew size is determined by the activity time assignment t_i , the decision variable for each activity.

For a given time assignment vector t let $CPM(t)$ denote the standard CPM scheduling computations of early (late) start and finish time and total activity slack. Consider $CPM(d)$. The computations determine the minimum project duration, T , and for each activity i , its slack, S_i . An activity i is *critical* if it is on a critical path, i.e., $S_i = 0$. If, on the other hand, $S_i > 0$ then activity i could operate for a longer period of time, using less resources per unit of time (if $i \in N$), without delaying the project beyond T .

Let a *slack path* be defined as a maximal length chain of slack activities. In general, the total slack of a given slack path can be allocated among the activities on that path whereby all these activities become critical. A time assignment vector that eliminates all slack from the *slack subnetwork*, i.e., the subnetwork of slack activities, is said to be *critical*. Let Λ_c be the set of critical time assignment vectors.

III. Resource Leveling

Given time assignment vector t and $CPM(t)$, a load history can be computed for each resource. (This assumes rounding of non-integer time assignments t .) If t is critical then $CPM(t)$ uniquely determines the resource load histories. If not, activity early (or late) starts, for example, could determine the schedule.

For $k \in K$, let $c_k(t)$ be the required capacity of resource k , defined as the maximum load over all time periods t , $1 \leq t \leq T$. Let p_k be the cost of maintaining a unit of capacity of resource k for the duration of the project.

We seek to minimize $\sum_{k \in K} p_k c_k(t)$ over Λ_c , the set of all critical time assignment vectors t .

In [2] this problem was recognized as combinatorially too complex, and a heuristic procedure was proposed in an effort to obtain a near optimal solution. This paper presents an improvement of the procedure.

A. Peak Pricing Procedure

A sequence of problems is considered in which only the activities operating in time periods where resource loads are "close" to capacity $c_k(t)$ are charged for their contribution to the capacity.

Let some time assignment vector t and the corresponding load histories be given. For each $k \in K$, define the peak intervals I_k as the set of time periods in which the load of resource k is within a specified percentage β of $c_k(t)$. Also, let I_k be the subset of I_k in which activity $i \in V$ operates. Of course, activity i may operate partially or completely outside I_k . Note that $I_k a_{ki}(t)$ is the amount of resource k applied to activity i in I_k . Finally, let $f_k \equiv |I_k|/|I_k|$ be the fraction of I_k in which activity $i \in V$ is active. (The dependence on t has been suppressed in the notation I_k, I_k, f_k .) In [2], the fraction f_k was approximated by a 0/1 coefficient, that is, $f_k = 1$ if and only if i operates in I_k .

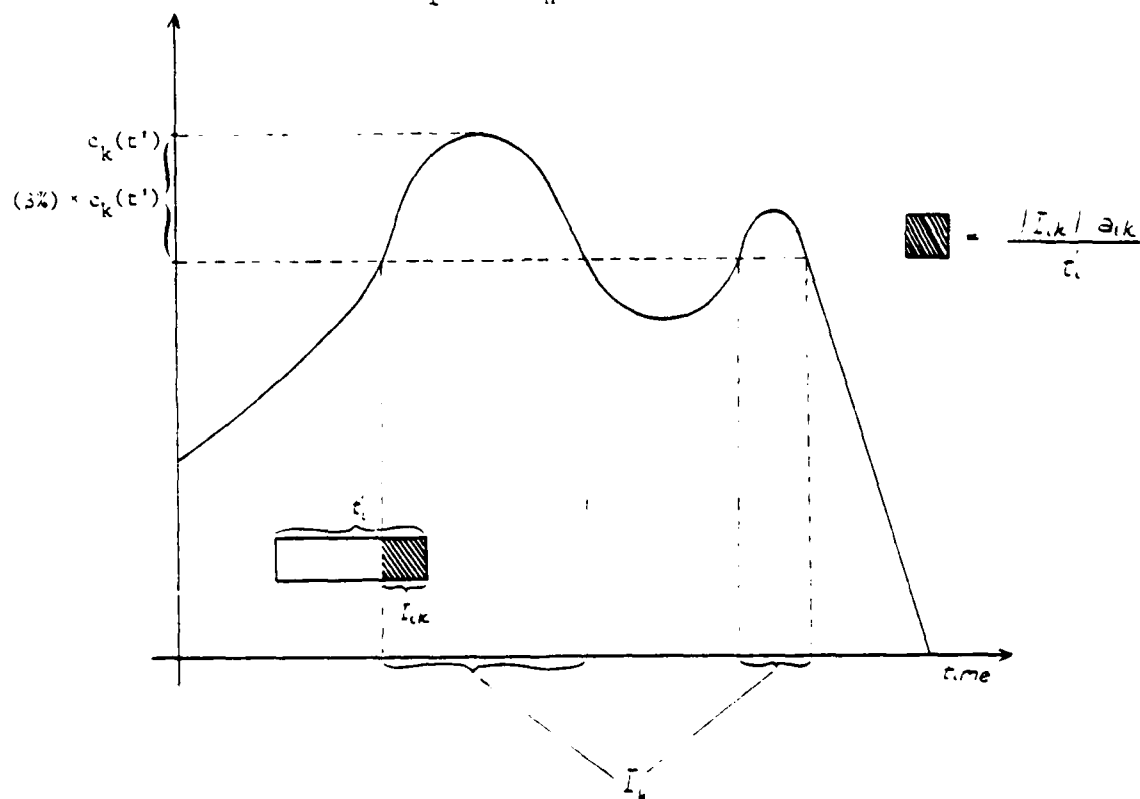
The capacity $c_k(t)$ can be approximated as follows:

$$\begin{aligned} \sum_{i \in V} \frac{f_k a_{ki}}{t} &= \frac{\sum_{i \in V} I_k a_{ki}(t)}{|I_k|} \\ &= \frac{\text{total amount of resource } k \text{ applied in } I_k}{\text{size of } I_k} \\ &\approx c_k(t). \end{aligned}$$

Exhibit 3 depicts the above relationships graphically.

EXHIBIT 3 RESOURCE PEAK INTERVALS

Load History of
Shop k (for assignments $t' = (t'_1, \dots, t'_n)$)




$c_k(t')$ = Peak load on Shop k for assignments t'

β = Peak interval tolerance

I_k = Peak intervals, Shop k

I_{ik} = Periods where activity i is in peak interval

 = Contribution of activity i to peak

If $f_{ik}(t)$, as a function of t , were readily computable we could perform

$$\min \left\{ \sum_{k \in K} p_k \sum_{i \in N} \frac{f_{ik}(t) a_{ik}}{t_i} : t \in \Lambda_t \right\}$$

to obtain an approximation of the minimum of $\sum_{k \in K} p_k c_k(t)$. This is not the case, so an iterative scheme is proposed.

Let some initial time assignment vector t' (for example d) be given.

- Step 1. Perform $CPM(t')$ and compute the resource load histories.
- Step 2. Compute $c_k(t')$ for all k . If no improvement has occurred in $\sum_{k \in K} p_k c_k(t')$, stop. Otherwise, determine I_k and f_{ik} for all $i \in N$, $k \in K$.
- Step 3. Solve

$$P_t: \min \left\{ \sum_{k \in K} p_k \sum_{i \in N} \frac{f_{ik} a_{ik}}{t_i} : t \in \Lambda_t \right\}.$$

Let t^* be an optimal solution to P_t .

Set $t' \leftarrow t^*$ and go to Step 1.

B. Optimization Problem Formulation

The mathematical program P_t can be formulated as a convex objective subject to linear constraints. Transformations of t allow Λ_t , the set of critical time assignment vectors, to be represented in two ways.

1. *Slack Allocation Variables:* Let $s \equiv t - d$, $s \equiv (s_1, \dots, s_q)$. Then s_i is the slack allocated to activity i given the time assignment $t \geq d$.

Let Λ_s be the set of slack allocation vectors s satisfying

$$C_s: \sum_{i \in \pi} s_i = \text{slack of path } \pi \quad \begin{array}{l} \text{for all slack paths } \pi \\ \text{in the slack subnetwork} \end{array}$$

$$s_i = 0 \quad \text{for all critical activities } i$$

$$s \geq 0.$$

Then $t \in \Lambda_t$ if and only if $s \in \Lambda_s$. Problem P_t becomes

$$P_s: \min \left\{ \sum_{k \in K} p_k \sum_{i \in N} \frac{f_{ik} a_{ik}}{s_i + d_i} : s \in \Lambda_s \right\}.$$

Let s^* be an optimal solution to P_s . Then $t^* = s^* + d$ is optimal for P_t . (This formulation was

presented in [2] with f_{jk} approximated as a 0/1 coefficient.)

The variables s_i for all critical activities i can be excluded from the problem. Furthermore, consider all slack paths which have only one element, i.e., an activity i by itself. If i does not occur in any other slack path then s_i can be set to its slack, S_i , thus eliminating a constraint and a variable. (See Exhibit 4a for an example of this.)

The number of linearly independent slack path constraints, n_{sp} , cannot exceed the number of slack allocation variables, n_{sa} , that remain, and may be considerably smaller (See Exhibit 4b).

The objective function coefficients in P_i (called *peak prices*) will be non-zero if and only if $\sum_{k \in K} f_{ik} a_{ik} > 0$.

Let $N(i)$ be the set of activities with non-zero peak prices, given time assignment vector i . Slack allocation variables for activities not in $N(i)$ can be eliminated by rewriting the constraint in C_i for slack path π as

$$\sum_{\substack{i \in \pi \\ i \in N(i)}} s_i \leq \text{slack of path } \pi.$$

C_i is thereby reduced in size. However, the slack of path π that is not allocated in the optimal solution of P_i , if non-zero, must somehow be allocated to the excluded activities on π to obtain a complete CPM schedule and thus derive the load histories. (These allocation decisions were made implicitly, and arbitrarily, in the original formulation P_i .) Scheduling the excluded activities could be done, for example, at early (or late) starts with minimum time assignments. The resulting load histories will certainly be influenced by these slack allocation decisions.

2. *Event Time Variables:* Let V be the set of network nodes (events) and let $(u(i), v(i))$ be the ordered pair of nodes representing arc (activity) i . Let $\tau \equiv (\tau_1, \dots, \tau_n)$ be an *event time vector*, where τ_v is the point in time at which node v is reached. Let Λ_i be the set of event time vectors satisfying

$$C_i: \tau_{v(i)} - \tau_{u(i)} \geq d_i \quad \text{for all } i \in A$$

$$\tau_v = \tau_{v'} \quad \text{for all nodes } v \in V \text{ on a critical path.}$$

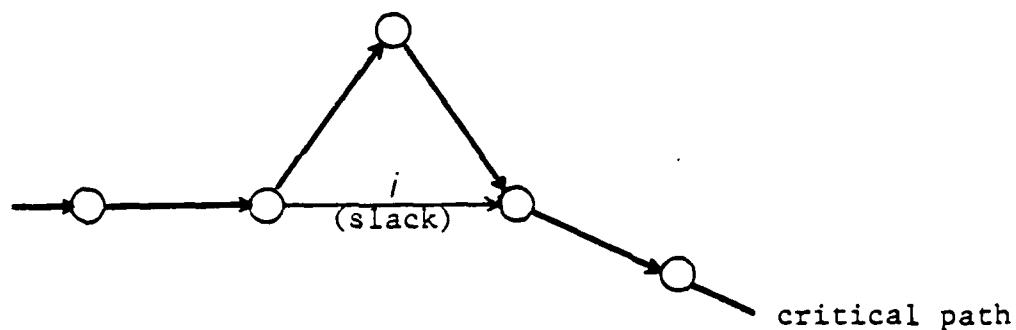
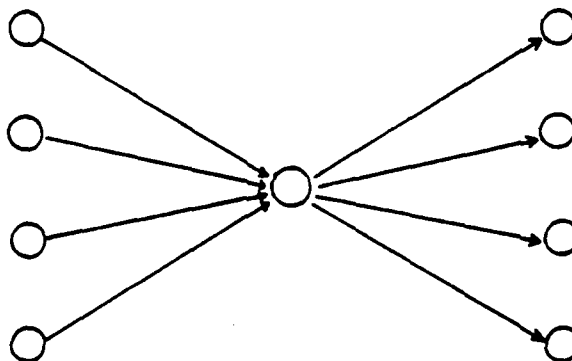
where $\tau_{v'}$ is the critical node (event) time of v , as derived from $CPM(d)$. C_i is similar to the standard CPM linear programming formulation. (See, for example, [3]). Then $i \in \Lambda_i$ if and only if $\tau \in \Lambda_i$. Problem P_i becomes

$$P_i: \min \left\{ \sum_{k \in K} p_k \sum_{i \in N} \frac{f_{ik} a_{ik}}{\tau_{v(i)} - \tau_{u(i)}} : \tau \in \Lambda_i \right\}.$$

EXHIBIT 4

(a) Single-Element Slack Paths

Activity i constitutes a "single-element slack path." Its slack can be pre-allocated.

(b) Redundant Slack Paths

This subnetwork of 8 slack activities gives rise to 16 slack path constraints (equalities), of which only 8 are linearly independent.

Let τ^* be an optimal solution to P_r . Then $t_i^* = \tau_{v(i)}^* - \tau_{u(i)}^*$ is optimal for P_r .

The event time variables τ_v for critical nodes v can be excluded from the problem. Furthermore, constraints for all activities i such that $u(i)$ and $v(i)$ are critical nodes (as shown in Exhibit 4a) become redundant.

C_r then consists of n_{SA} inequalities in n_{SV} variables, where n_{SV} , the number of slack nodes, is usually considerably smaller than n_{SA} . Also, $n_{SV} < n_{SP}$.

In its most compact form, C_r consists of n_{SP} inequalities in $|N(i)|$ variables, plus non-negativity constraints. Exhibit 5 displays the constraints sets. The use of P_r or P_r will depend on the way in which the linear constraint set is imbedded in the non-linear programming algorithm. If the algorithm uses the dual constraint set in an imbedded linear program (see, for example, [1]) then P_r will most likely have the LP basis of smallest size.

C. Procedure Termination and Cycling

The allocation of remaining slack for activities with zero peak prices is made arbitrarily in both formulations. This is a symptom of the inherent weakness of the Peak Pricing Procedure. Since f_{ik} does not change within an iteration as the time assignments (or their transformations) do, slack is allocated without feedback on the resulting shift in resource loads. A procedure can be imagined where feedback is more frequent, at the price of more frequent load history recomputations.

In general, no guarantee of improvement in each iteration of the procedure, let alone convergence to optimality, can be given. The new resource peaks will tend to be outside the previous peaks and may even be higher than before. In our experience a commonly occurring situation was the cycling between two different sets of peak loads. The following modification to the procedure was made to cope with this problem.

Given time assignment vectors t^1, t^2, \dots, t^L from previous iterations, define

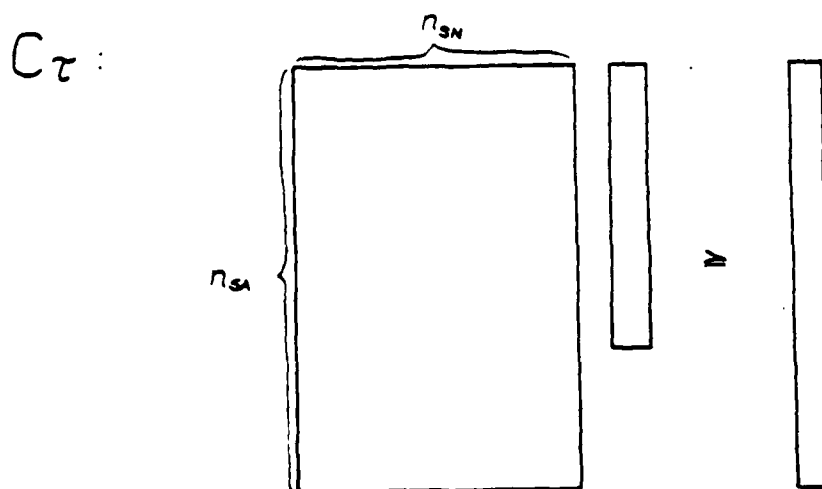
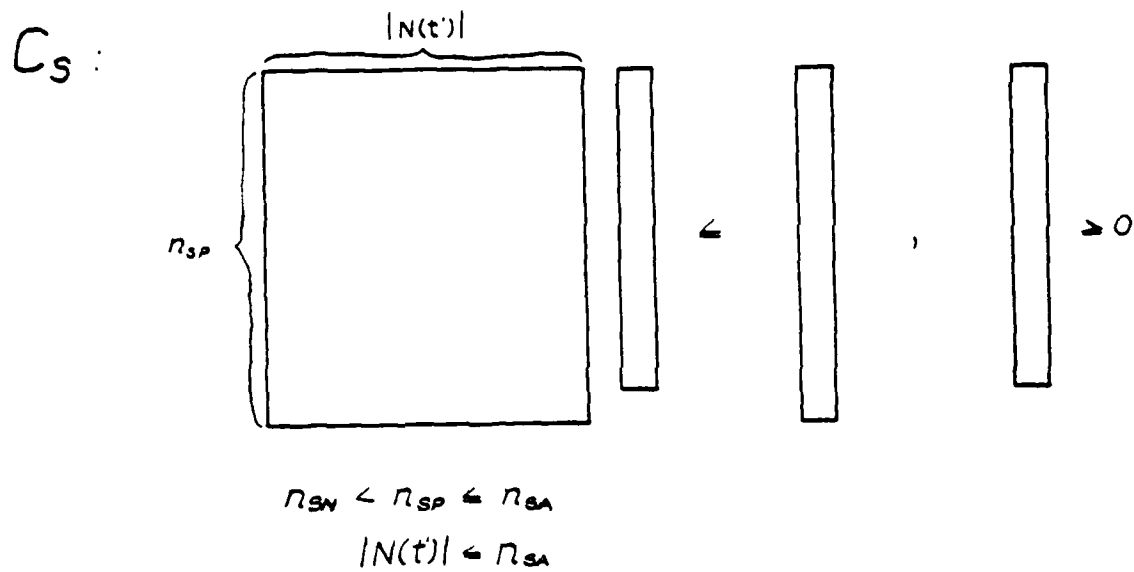
$$f_k(t^1, \dots, t^L) \equiv \sum_{l=1}^L \alpha_{kl} f_{ik}(t^l), \quad \text{where } \alpha_{kl} \geq 0, \sum_{l=1}^L \alpha_{kl} = 1.$$

An obvious choice for α_{kl} is

$$\alpha_{kl} = \frac{c_k(t^l)}{\sum_{l=1}^L c_k(t^l)},$$

which assigns weights to resource capacities according to their relative contribution over all L iterations.

EXHIBIT 5
OPTIMIZATION PROBLEM CONSTRAINT SETS



The coefficients f_{ik} depend of course on the definition criterion for I_k . The criterion could vary, for example, from resource to resource, or from iteration to iteration, depending on the particular resource load pattern. Some experience with different criteria is described below.

D. Capacity Requirements vs. Project Delay

The formulations P_1 and P_2 can be adapted to study the trade-off between resource capacity cost and project delay. Let $\gamma(T)$ be some project opportunity cost as a function of the variable project duration T . For simplicity of exposition, we assume x to be the only starting node and y the only finishing node in the network.

P_1 becomes

$$\begin{aligned} \min \sum_{k \in K} p_k \sum_{i \in V} \frac{f_{ik} a_{ik}}{s_i + d_i} + \gamma(T) \\ \text{s.t. } \sum_{i \in \pi} s_i = T - \sum_{i \in \pi} d_i \quad \text{for all paths } \pi \text{ from } x \text{ to } y \\ \text{in the network} \\ s_i \geq 0, \end{aligned}$$

and P_2 becomes

$$\begin{aligned} \min \sum_{k \in K} p_k \sum_{i \in V} \frac{f_{ik} a_{ik}}{\tau_{v(i)} - \tau_{u(i)}} + \gamma(T) \\ \text{s.t. } \tau_{v(i)} - \tau_{u(i)} \geq d_i \quad \text{for all } i \in A \\ \tau_v - \tau_x = T. \end{aligned}$$

Since the durations of previously critical activities are no longer fixed, the variables and constraints associated with them can no longer be excluded a priori.

IV. Applications and Computational Experience

A. Network and Data

The original key-op network for a naval ship overhaul with about 1200 key-ops using 20 different labor types (resources) was selected for testing of the new techniques. Parallel or series key-ops that had similar work content (labor type) and dealt with the same technical subsystem, e.g., removal of various components of the main steam system, were grouped to form one aggregate activity. Attention was paid to combine key-ops with similar resource application rates. In this manner, a more aggregated network was developed, consisting of 316 activities using resources and 75 other activities (milestones, dummy activities, time lag activities). The largest twelve labor shops were retained with a total of 32,600 man-days of labor requirements. See Exhibit 6 for a list of shops and the labor requirements per shop. See Exhibit 7 for an example of an aggregate activity.

The minimum duration of each aggregate activity was computed as the maximum of the path lengths of the key-op activities which comprised the aggregate activity. The project duration was 211 days.

In the absence of good capacity cost estimates, the cost coefficients p_k were set to 1 and the objective became to minimize the sum of the shop capacity requirements.

The shops interact quite strongly. This is most evident in the case of shops 4 and 5. Shop 5 is responsible for removal, re-installation, and testing of mechanical subsystems. Shop 4 performs the on-shore repair of the subsystem components. The load pattern of shop 5 shows two regions of peak labor requirements: one during the removal phase, the second during re-installation. The peak of shop 4 lies in the valley between shop 5's peaks. (See Exhibit 8.)

The slack path formulation of the optimization step was chosen. To reduce the size of the problem those activities whose labor requirements per shop per day (a_k/d_k) did not exceed one man-day were excluded from the optimization problem P_0 . It was reasoned that slack allocation to these activities would have reduced resource capacities only insignificantly. Ninety-eight activities with 3,600 man-days of labor requirements (11% of total requirements) were thus excluded. The constraint set C_0 was reduced to 170 inequalities in 168 variables. An accelerated feasible conjugate direction algorithm due to [1] was used to solve the problems P_k .

B. Experiments

(a) Early experiments were performed with the 0/1 coefficients f_k and various peak interval thresholds β ranging from 2% to 15%. Sequences of iterations with β varying from iteration to iteration led to inconclusive results. A value of $\beta=5\%$ was then chosen for experiments

EXHIBIT 6
LIST OF LABOR SHOPS

Numbers	Name	Project Labor Requirements (man-days)
1	Structural Group I	1330
2	Structural Group II	1390
3	Weld & Burn	2290
4	Mechanical Group-Shop	4310
5	Mechanical Group-Shipboard	5110
6	Boilermakers	2310
7	Electrical	3040
8	Pipe-fitting	5910
9	Electronics	2800
10	Shipwright	640
11	Painting	3000
12	Rigging	540
	Total Project Labor Requirements	<hr/> 32670

EXHIBIT 7

EXAMPLE OF AGGREGATE ACTIVITY

Aggregate Activity: Inspect and Remove Fuel Oil System Components

Principal Shops Involved: Shop 8 - Pipefitting

Total Labor Requirements: 93 Man-days

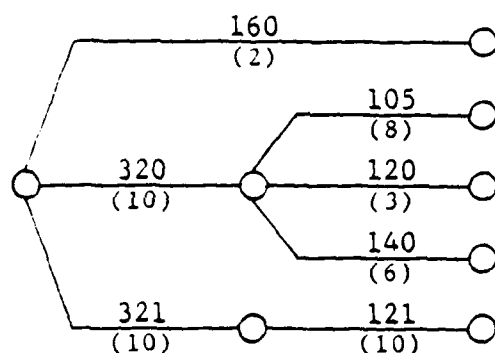
Minimum Duration: 20 days

Component Key-ops:

Key-op No.	Name	Minimum duration (days)	Total Labor requirements* (man-days)
105	Inspect Piping	8	10
120	Remove Relief Valves	3	2
121	Remove Strainers	10	30
140	Remove Pumps	6	8
160	Remove Controllers	2	2
320	Flush System	10	20
321	Flush Strainers	10	30
			<hr/> 93

*summed over all shops

Key-op Subnetwork



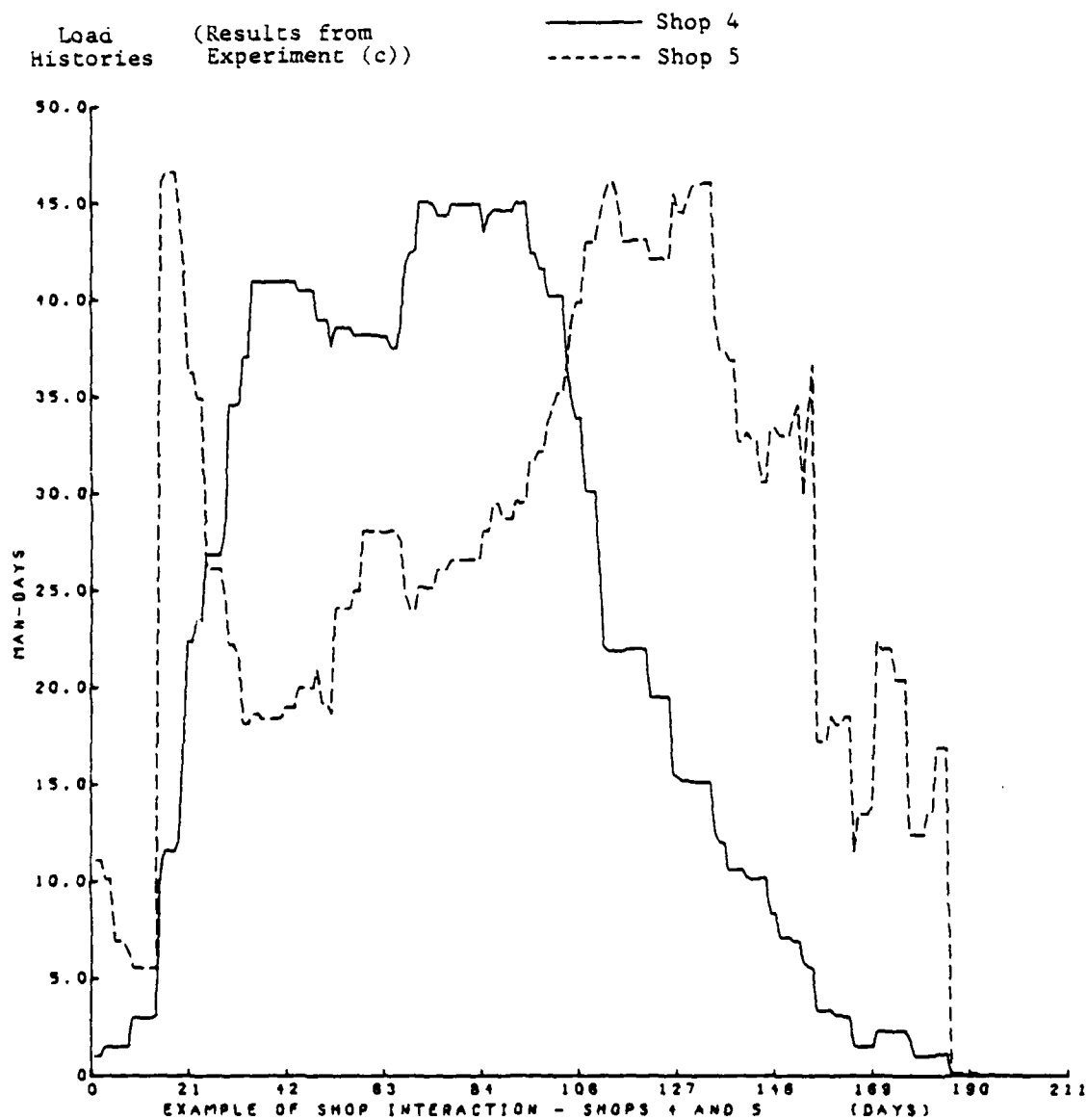
Legend:

(2) - minimum duration
160 - key-op number

longest path

= minimum duration for aggregate activity

EXHIBIT 8
EXAMPLE OF SHOP INTERACTION



(b) and (c).

(b) Initial load histories were computed from $CPM(d)$ with an early start schedule. Then the slack of all single-element paths was allocated (as described in Section III.B.), which yielded an objective improvement of 12%. The improved definition of f_k was used to compute peak prices for subsequent iterations of the Peak Pricing Procedure. Initially, four iterations were performed in succession. Pairs of successive iterations showed cycling behaviour. Multiple iteration weighting using iterations 1 and 2, and 3 and 4, was tried. The iteration with weighted peak prices from iterations 1 and 2 produced the best results thus far, an improvement of 31% over the initial solution. (See Exhibits 9 and 10.) Further iterations and weightings did not improve the solution.

(c) To test the effect of different initial load histories, a late start schedule from $CPM(d)$ was used with load patterns markedly different from the early start schedule. As before, single-element path slack was pre-allocated. The first iteration produced a solution very close to the best result of (b). Individual shop capacities are similar in size (see Exhibit 9) and the load patterns have similar shape (See Exhibit 11 for the case of shop 8.)

(d) By setting $\beta=100\%$, the objective function of P_1 becomes

$$\sum_{k \in K} p_k \sum_{e \in A} \frac{a_{ek}}{s_e + d_e} .$$

so that P_1 can now be stated as minimizing the sum of the resource application rates over all activities. The solution obtained was quite good compared with (b) and (c). (See Exhibit 9 for labor requirements by shop, and Exhibit 11 for a comparison of load patterns of shop 8.)

EXHIBIT 9

EXPERIMENTAL RESULTS

Shop Capacity Requirements (Man-days)														Number of NLP Iter. *	Number of LP Iter. *	CPU Time (sec.) **
1	2	3	4	5	6	7	8	9	10	11	12	Total				
<u>Experiment (b)</u>																
Early Start Schedule																
	23	17	35	66	67	45	33	60	26	14	55	9	450			
After Allocation of Slack of Single-Element Slack Paths																
	18	14	32	64	61	41	31	50	22	9	46	8	396			
After Iteration 1																
	13	11	24	47	64	21	25	47	18	9	41	7	327	14	251	21.8
After Iteration 2																
	13	11	23	49	53	38	24	47	18	9	42	7	334	7	143	11.1
After Weighting Peaks of Iterations 1 and 2																
	13	11	23	47	48	24	25	42	18	8	41	7	308	6	165	14.7
<u>Experiment (c)</u>																
Late Start Schedule																
	21	18	33	64	79	36	38	69	21	12	44	7	442			
After Allocation of Slack of Single-Element Slack Paths																
	17	16	29	62	73	37	32	57	21	9	35	6	394			
After Iteration 1																
	13	11	23	45	47	25	25	44	20	8	40	7	308	19	198	16.0
<u>Experiment (d)</u>																
Beta = 100%																
	13	12	22	47	54	27	25	43	19	8	40	7	317	13	453	40.9

* Feasible Conjugate Direction Algorithm with Imbedded LP [1]
 ** CDC 6400

EXHIBIT 10
Experiment (b)
SEQUENCE OF LOAD HISTORIES-SHOP 5

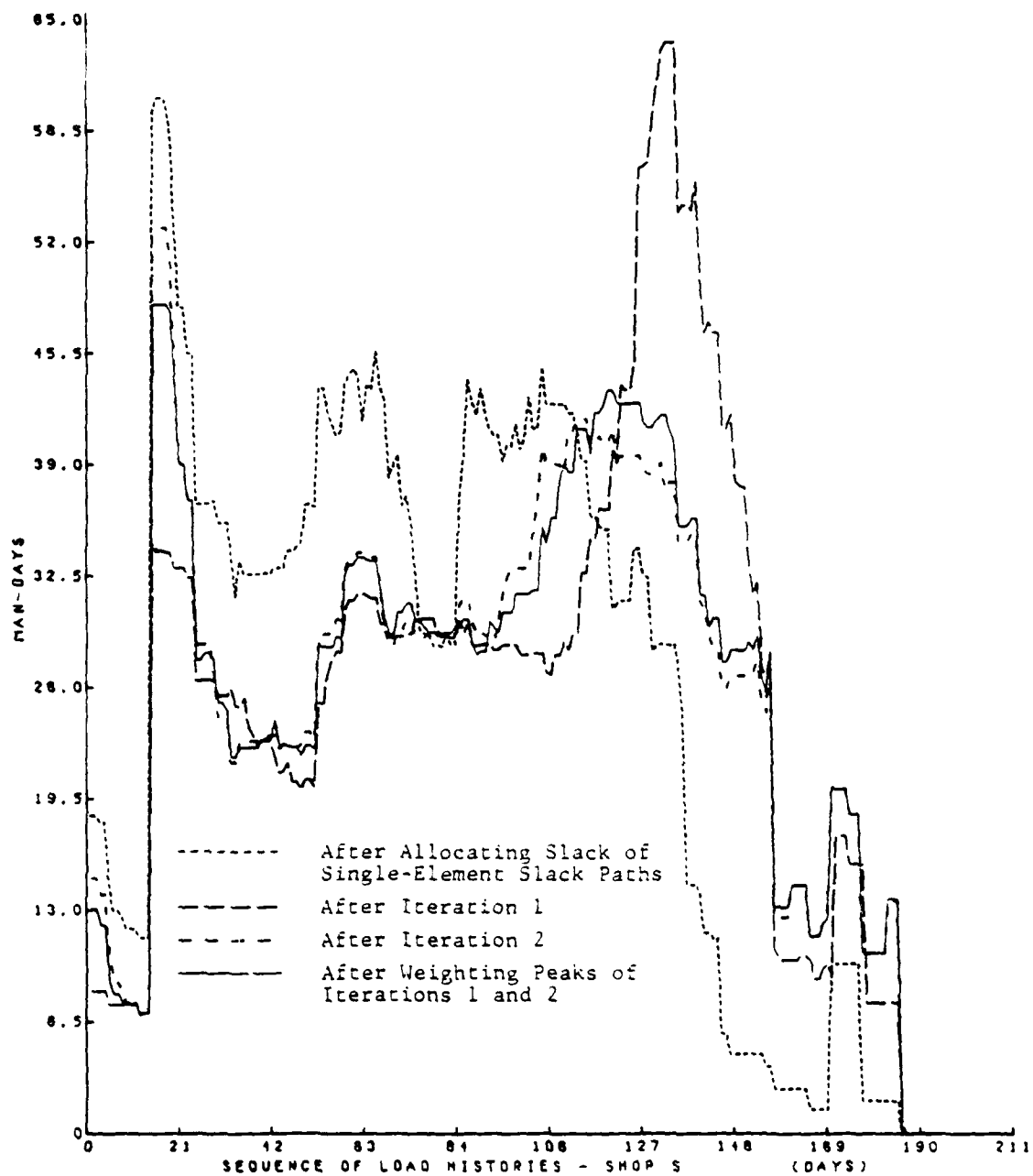
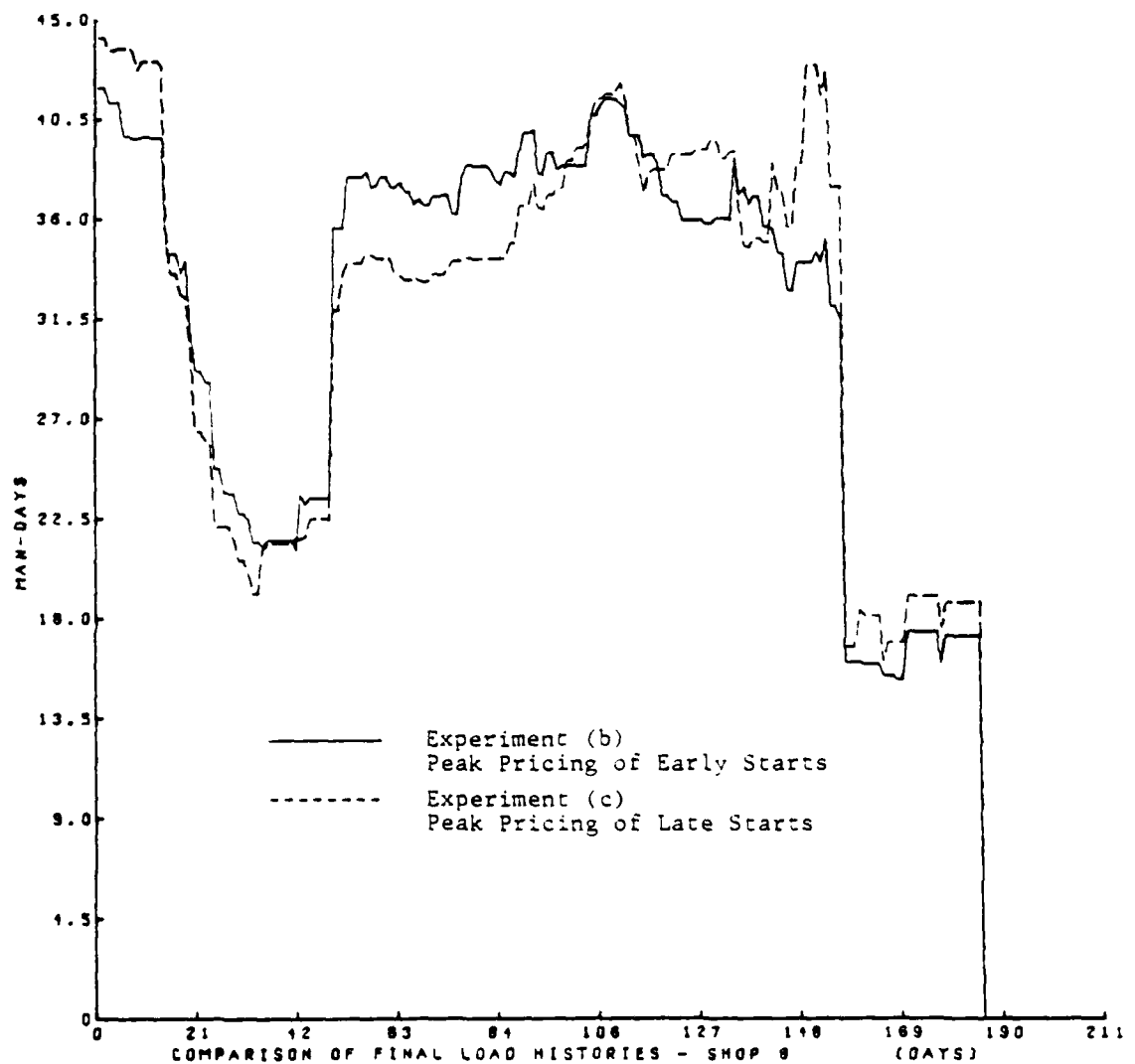


EXHIBIT 11
COMPARISON OF SHOP 8 LOAD HISTORIES



V. Summary and Analysis of Results

The development of new techniques for leveling resource loads of individual overhaul projects in a shipyard has been reviewed. The techniques exploit the crew size flexibility available to management in selecting time durations for aggregated activities.

Careful development of the overhaul data is mandatory. Activity labor requirements cannot be estimated solely from historical data, but depend on the specific work involved. Historical activity durations will certainly not represent minimum time assignments. Due to the assumption of constant activity loading, the minimum time assignments, together with the activity labor requirements, determine the maximum crew size assignable to each activity. Known crew size constraints may actually be the basis for the minimum time assignment estimates.

While optimal solutions cannot be generated, considerable loading improvement of initially selected schedules can be made. The effect of varying peak interval thresholds and multiple iteration weighting is not completely understood. Their effect will depend on the structure of the network. This is most likely the reason for the differences in performance between experiments (b) and (c).

Elimination of all slack from the network obviously increases the risk of project slippage. Additional constraints can be included to retain some amount of slack between milestones. In some cases, the final activity time assignments were unreasonably large. Time assignment upper bounds could remedy the problem of overstretched activities.

Varying activity durations (replanning) is an alternative to varying activity starting dates (rescheduling) as a means of resource leveling. However, replanning is not proposed as a complete substitute for rescheduling, in fact some rescheduling of the best replanning solutions obtained did improve resource loading, albeit slightly. Nonetheless, we submit that replanning of activities is a powerful and useful technique for leveling resource peaks in shipyards and related project industries.

Glossary of Notation

A	- set of activities
a_{ik}	- labor requirement of activity i , shop k
α_{kl}	- peak price weighting factor
β	- peak interval threshold percentage
C	- constraint set of P
$c_k(t')$	- required capacity of shop k given time assignment t'
$CPM(t')$	- CPM schedule computations given t'
d_i	- minimum time assignment of activity i , $d \equiv (d_1, \dots, d_{ A })$
f_{ik}	- fraction of I_k in which activity i operates
I_{ik}	- subset of I_k in which activity i operates
I_k	- set of time periods within β of $c_k(t')$
K	- set of resources
Λ	- set of critical time assignment vectors
N	- set of activities with non-zero resource use
$N(t')$	- set of activities with non-zero peak prices, given t'
n_{SA}	- number of slack activities
n_{SN}	- number of slack nodes
n_{SP}	- number of slack paths
P	- optimization problem
p_k	- unit capacity cost for resource k
S_i	- slack of activity i , given $CPM(d)$
s_i	- slack allocation to activity i , $s \equiv (s_1, \dots, s_{ A })$
T	- project duration
t_i	- time assignment of activity i , $t \equiv (t_1, \dots, t_{ A })$
τ_v	- event time of node v , $\tau \equiv (\tau_1, \dots, \tau_{ V })$
τ_v^*	- event time of critical node v , given $CPM(d)$
$(u(i), v(i))$	- node pair associated with activity i
V	- set of network nodes

References

- [1] Best, Michael J. and Ritter, Klaus. "An Accelerated Conjugate Direction Method to Solve Linearly Constrained Minimization Problems." *Journal of Computer and Systems Sciences*, Vol. 11, No. 3 (1975).
- [2] Leachman, Robert C., "Aspects of Dynamic Production Planning," *Report ORC 79-8*, Operations Research Center, University of California, Berkeley (1979).
- [3] Moder, Joseph J. and Phillips, Cecil R., *Project Management with CPM and PERT*, Second Edition, Van Nostrand and Reinhold, New York (1970).
- [4] Shephard, Ronald W., Al-Ayat, Rokaya, and Leachman, Robert C., "Shipbuilding Production Function: An example of a Dynamic Production Function," *Report ORC 76-25*, Operations Research Center, University of California, Berkeley (1976).

**DAT
FILM**